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Quasielastic electron scattering in relativistic mean-field theory

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Abstract. The density-dependent relativistic hadron (DDRH) field theory proposed recently is extended to investigate the longitudinal response function and the Coulomb sum rule in quasielastic electron scattering in the relativistic random phase approximation (RPA). The results in the DDRH model are compared with those in other models systematically. It is found that meson effective masses induced by the nonlinear terms in the nonlinear Walecka model should be used to obtain the meson Green's functions when the longitudinal response function and the Coulomb sum rule are calculated. The effects of the δ and ρ mesons are clearly shown in quasielastic electron scattering, and the isospin-dependent attractive potential between nucleons due to the exchange of the δ-meson cancels the isospin-dependent repulsive contribution of the ρ-meson to a certain extent. The obtained results in the DDRH model are in good agreement with experimental data except for the Coulomb sum rule in ^{208}Pb.

PACS. 25.30.Fj Inelastic electron scattering to continuum – 21.65.+f Nuclear matter – 21.10.-k Properties of nuclei; nuclear energy levels – 21.60.-n Nuclear structure models and methods

1 Introduction

Relativistic mean-field models based on nucleons and mesons as their effective degrees of freedom are successful in describing nuclear matter and finite nuclei [1]. The Walecka-I model contains fields for nucleons and scalar σ and vector ω mesons. The Walecka-II model is an extension of the Walecka-I one with the inclusion of the isovector ρ-meson. Since the Walecka-I (II) model gives too large a value of nuclear-matter incompressibility at the saturation density, in order to overcome this shortcoming of the Walecka-I (II) model, the nonlinear σ or ω terms were introduced into the Walecka-I (II) model, and the so-called nonlinear Walecka model was constructed [2]. With the model parameters calibrated to the many-body properties of nuclear matter at the saturation density, the nonlinear Walecka model has been applied to nuclear matter and finite nuclei quite successfully [3].

The density-dependent relativistic hadron (DDRH) field theory was proposed recently by Hofman, Keil, and Lenske [4], where, in addition to the σ, ω, and ρ mesons, the isovector-scalar δ-meson is included. Meanwhile, the meson-nucleon vertices in the DDRH model are determined in such a way that the scalar and vector potentials reproduce those of the Dirac-Brueckner (DB) calculation at each density of nuclear matter exactly. In this way, the analytic density-dependent meson-nucleon vertices in the DDRH model have been obtained. Since the interaction in the DB theory is determined by fittings to NN-scattering data, the parametrization of the DDRH model is independent of any phenomenological fit to data of the nuclear many-body effect, *i.e.*, the feature of the DDRH model is no free parameters, and has a solid foundation for relativistic nuclear many-body physics on the microscopic level. Intensive investigations have shown that the DDRH model can be used to describe exotic nuclei [4], the properties of neutron star matter [5], kaon condensation in dense matter [6,7], etc., and becomes one of the successful relativistic nuclear models.

Quasielastic electron scattering is a useful approach to probe the properties of nucleons in a nucleus, and provides one of the feasible ways to examine in detail theoretical nuclear models. The linear Walecka-I (σ, ω) [8], and Walecka-II (σ, ω, ρ) models [9] have been applied to study the nuclear response function in quasielastic electron scattering in the local density approximation [10]. Also, the calculations of the longitudinal response function in quasielastic electron scattering at a relatively high momentum transfer have been performed recently in the nonlinear Walceka model [11]. However, how the density-dependent relativistic nuclear models used quite successfully in nuclear matter and finite nuclei, such as the DDRH model and the TW model, describe quasielastic electron scattering is still lacking.

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In this paper, the DDRH model is extended to investigate the target nuclear-structure effects on quasielastic electron scattering in the relativistic random phase approximation (RPA). The results in the DDRH model will be compared with those in the density-dependent relativistic nuclear model developed by Typel and Wolter (also referred to as the TW model) [12], and the linear and nonlinear Walecka models [1,3]. The roles of the nonlinear terms in the nonlinear Walecka model as well as the δ and ρ mesons will be discussed.

The rest of this paper is organized as follows. In sect. 2, a brief review of the models and the basic formulae are given. The results and discussion are presented in sect. 3.

2 The formulae

The Lagrangian densities in the various relativistic nuclear models can be unified as

$$\begin{aligned}\mathcal{L} = &\ \overline{\psi}\Big[\gamma_\mu\Big(i\partial^\mu - \Gamma_\omega\omega^\mu - \frac{\Gamma_\rho}{2}\vec{\tau}\cdot\vec{\rho}^\mu - e\frac{1}{2}(1-\tau_3)A^\mu\Big) \\ &- \Big(M - \Gamma_\sigma\sigma - \Gamma_\delta\vec{\tau}\cdot\vec{\delta}\Big)\Big]\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ &+ \frac{1}{2}\Big(\partial_\mu\vec{\delta}\cdot\partial^\mu\vec{\delta} - m_\delta^2\vec{\delta}\cdot\vec{\delta}\Big) - \frac{1}{3!}\kappa\sigma^3 - \frac{1}{4!}\lambda\sigma^4 \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4!}\xi(\omega_\mu\omega^\mu)^2 \\ &- \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \frac{1}{4}\mathbf{A}_{\mu\nu}\mathbf{A}^{\mu\nu}\,, \end{aligned} \tag{1}$$

where ψ stands for the nucleon field, A_μ is the photon field. σ, ω_μ, $\vec{\delta}$, and $\vec{\rho}_\mu$ are for σ-, ω-, δ-, and ρ-meson fields. Here $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, and $\mathbf{A}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The double-differential electron scattering cross-section is expressed as [9]

$$\begin{aligned}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E} = \sigma_\mathrm{M}\Bigg\{&\left(\frac{q^2}{|\mathbf{q}|^2}\right)^2 S_\mathrm{L}(q) \\ &+ \left[\frac{-q^2}{2|\mathbf{q}|^2} + \tan^2\left(\frac{\theta}{2}\right)\right] S_\mathrm{T}(q)\Bigg\}\,,\end{aligned} \tag{2}$$

where the σ_M is the Mott cross-section, $q = (\omega, |\mathbf{q}|)$ is the four-momentum transfer of the electron to a target nucleus, S_L and S_T are the longitudinal and transverse response functions. The polarization tensor as the ground-state expectation value of a time-ordered product of electromagnetic current operator is

$$i\Pi^{\mu\nu}(q) = \int \mathrm{d}^4x e^{iqx}\langle\psi|\mathrm{T}(\hat{J}^\mu(x)\hat{J}^\nu(0))|\psi\rangle\,. \tag{3}$$

The electromagnetic current operator $\hat{J}^\mu(q)$ is defined as

$$\begin{aligned}\hat{J}^\mu(q) = &\int \mathrm{d}^3x e^{i\mathbf{q}\cdot\mathbf{x}}\hat{\bar{\psi}}(\mathbf{x}) \\ &\times\Big(F_1(q^2)\gamma^\mu + F_2(q^2)\frac{\kappa_\tau}{2M}i\sigma^{\mu\nu}q_\nu\Big)\hat{\psi}(\mathbf{x})\,,\end{aligned} \tag{4}$$

where $\hat{\bar{\psi}}$ and $\hat{\psi}$ are nucleon field operators, M is the nucleon bare mass. $\sigma_{\mu\nu} \equiv \frac{1}{2}i[\gamma_\mu, \gamma_\nu]$. The proton and neutron anomalous magnetic moments are $\kappa_p = 1.793$ and $\kappa_n = -1.913$. The Dirac and Pauli form factors for nucleons $F_1(q^2)$ and $F_2(q^2)$ are given by [13]

$$\begin{aligned}F_{1p}(q^2) = &\ F_{2n}(q^2)\left[\frac{1-(q^2/4M^2)(1+\kappa_p)}{1-q^2/4M^2}\right]\,,\\ F_{2p}(q^2) = &\ F_{2n}(q^2)\left(\frac{1}{1-q^2/4M^2}\right)\,,\\ F_{1n}(q^2) = &\ 0\,,\\ F_{2n}(q^2) = &\ \left(1-\frac{q^2}{0.71\,\mathrm{GeV}^2}\right)^{-2}\,.\end{aligned} \tag{5}$$

The longitudinal part $\Pi_\mathrm{L}^\mathrm{RPA}$ of the meson polarized self-energy induced by the electromagnetic current operator $\hat{J}^\mu(q)$ in the RPA can be written as

$$\Pi_\mathrm{L}^\mathrm{RPA} = \Pi_\mathrm{L}^\mathrm{H} + \delta\Pi_\mathrm{L,is}^\mathrm{RPA} + \delta\Pi_\mathrm{L,iv}^\mathrm{RPA}\,, \tag{6}$$

where $\delta\Pi_\mathrm{L,is}^\mathrm{RPA}$ and $\delta\Pi_\mathrm{L,iv}^\mathrm{RPA}$ stem from $\hat{J}_\mathrm{is}^\mu(q)$ and $\hat{J}_\mathrm{iv}^\mu(q)$ which represent the isoscalar and isovector parts of $\hat{J}^\mu$, respectively. The formulae of $\Pi_\mathrm{L}^\mathrm{H}$ and $\delta\Pi_\mathrm{L,is}^\mathrm{RPA}$ can be derived as in ref. [10]. Similarly, we can obtain the formulae of $\delta\Pi_\mathrm{L,iv}^\mathrm{RPA}$, in which the contributions of the isovector (ρ) meson and the isoscalar (δ) meson are considered in the DDRH model.

In the local density approximation, the relation between the longitudinal response function $S_\mathrm{L}(q)$ and the longitudinal part of polarized self-energy $\Pi_\mathrm{L}(q)$ is

$$S_\mathrm{L}(q) = \frac{1}{\pi}\int \mathrm{d}^3r\mathrm{Im}\Pi_\mathrm{L}(q)\,. \tag{7}$$

Obviously, $\mathrm{Im}\Pi_\mathrm{L}(q)$ in a target nucleus depends on the space coordinate r because the proton and neutron densities $\rho_{p,n}(r)$, and the nucleon effective mass $M_{p,n}^*(r)$ are r-dependent. $\rho_{p,n}(r)$ and $M_{p,n}^*(r)$ in a target nucleus can be obtained consistently by solving nonlinear coupled differential equations for finite nuclei in relativistic mean-field approximation. The detailed numerical techniques and basic formulae for finite nuclei are given in ref. [4] for the DDRH model, [12] for the TW model, [1] for the linear Walecka model, and [3] for the nonlinear Walecka model. The extracted r-dependent physical quantities are then used to calculate the longitudinal response function and the Coulomb sum rule. It should be noted that the effects of finite nuclei entered fundamentally in this way are parameter free. Meanwhile the dynamical properties for each model are taken into account completely.

The Coulomb sum rule is defined as [8]

$$C(|\mathbf{q}|) = \int_0^{|\mathbf{q}|}\mathrm{d}\omega S_\mathrm{L}(q)\,. \tag{8}$$

3 Results and discussion

Firstly, we discuss the roles of the nonlinear terms in the nonlinear Walecka model with Tm1 as a representative

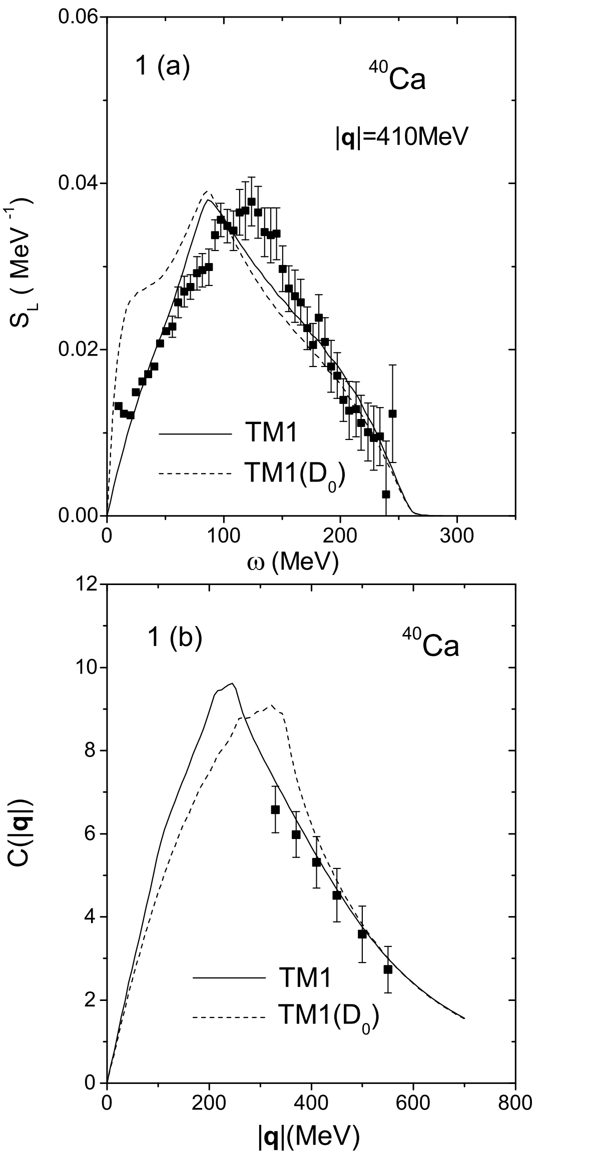

Fig. 1. Longitudinal response function (panel 1(a)) and Coulomb sum rule (panel 1(b)) for ^{40}Ca. Experimental data are taken from ref. [14].

parameter set. The nonlinear terms affect not only the local physical quantities $\rho_{p,n}(r)$ and $M^*_{p,n}(r)$ for relativistic mean-field calculations of a target nucleus, but also the meson Green's functions. Normally, one of the key steps in analyzing quasielastic electron scattering is to use the meson bare masses to calculate the meson Green's functions in the relativistic mean-field approximation. However, it has been shown in our calculations that the meson effective masses instead of the meson bare ones should be used to calculate the meson Green's functions in the nonlinear Walecka model. This process can be carried out by replacing the meson bare masses m_σ^2 and m_ω^2 by the meson effective ones $m_\sigma^{*2} = m_\sigma^2 + \kappa\sigma_0 + \frac{1}{2}\lambda\sigma_0^2$ and $m_\omega^{*2} = m_\omega^2 + \frac{1}{2}\xi V_0^2$, where σ_0 and V_0 are the expectation values of σ- and ω-meson fields in a nucleus ground state. For detailed comparisons we have plotted in fig. 1 the longitudinal response function at $q = 410$ MeV and the Coulomb sum rule with the meson bare (labelled by Tm1(D_0)) and effective (labelled by Tm1) masses for ^{40}Ca. It is evident in fig. 1(a) that the curve with Tm1(D_0) has a strange bump at $q \sim 25$ MeV. Furthermore, the trend of the curve with Tm1(D_0) in fig. 1(b) does not follow the experimental data. These results illustrate that the curve with Tm1(D_0) is unrealistic in comparison with that with Tm1 since the meson bare masses (*i.e.* not the meson effective ones) are used to calculate the meson Green's functions for the results with Tm1(D_0). The shortcoming of the results with Tm1(D_0) can also be understood from the value of the Landau parameter F_0 which must satisfy $F_0 > -1$ [15]. For $M^*/M = 0.64$ at the saturation density of nuclear matter in the nonlinear Walecka model, we obtain $F_0 = -2.75 < -1$ for the results with Tm1(D_0), while $F_0 = -0.07 > -1$ for the results with Tm1. The former case is unreasonable, and leads to the unstability of nuclear matter, thus we will give up the results with Tm1(D_0) in our following calculations.

The results in the various models at $|\mathbf{q}| = 410$ MeV and $|\mathbf{q}| = 550$ MeV for ^{40}Ca, ^{48}Ca, and ^{56}Fe are presented in figs. 2, 3, and 4, and those at $|\mathbf{q}| = 400$ MeV and $|\mathbf{q}| = 550$ MeV for ^{208}Pb in fig. 5. For ^{40}Ca, ^{48}Ca, and ^{56}Fe it has been shown that the curves of the longitudinal response function and the Coulomb sum rule in the various models have similar shapes. At $|\mathbf{q}| = 410$ MeV (see figs. 2(a), 3(a), and 4(a)), the maximum values of the curves occur at a similar energy transfer, and shift to the left side compared with experimental data. While at $|\mathbf{q}| = 550$ MeV (see figs. 2(b), 3(b), and 4(b)) the maximum values of the curves are approximately located at the same energy transfer as experimental data. These results show that quasielastic electron scattering can be described better in the relativistic mean-field theory at higher momentum transfer than at lower one. The results given by the DDRH model have a slightly higher peak than those of other models. For the Coulomb sum rule (see figs. 2(c), 3(c), and 4(c)) it is apparent that the peak of the DDRH model is also higher than those of other models. For ^{208}Pb in fig. 5, the results of the longitudinal response function are in good agreement with experimental data. However, the results of the Coulomb sum rule are not reproduced well at high momentum transfers, which were also shown in ref. [16]. Considering the properties of isospin asymmetry in ^{208}Pb (*i.e.*, $N = 82$, and $Z = 126$), the isovector-vector (ρ) and isovector-scalar (δ) mesons are included in the DDRH model in our calculations compared with those in ref. [16], the discrepancy with experimental data for the Coulomb sum rule in ^{208}Pb still remains. These results imply that the Coulomb sum rule in a heavy nucleus like ^{208}Pb is not explained well in relativistic mean-field theory.

The most important difference between the DDRH model and other models is that the DDRH model contains the δ-meson. In order to discuss the roles of the δ-meson we have plotted, as an example, the longitudinal response function at $|\mathbf{q}| = 410$ MeV and the Coulomb sum rule for ^{48}Ca in figs. 6(a) and (b). It is seen in fig. 6(a) that

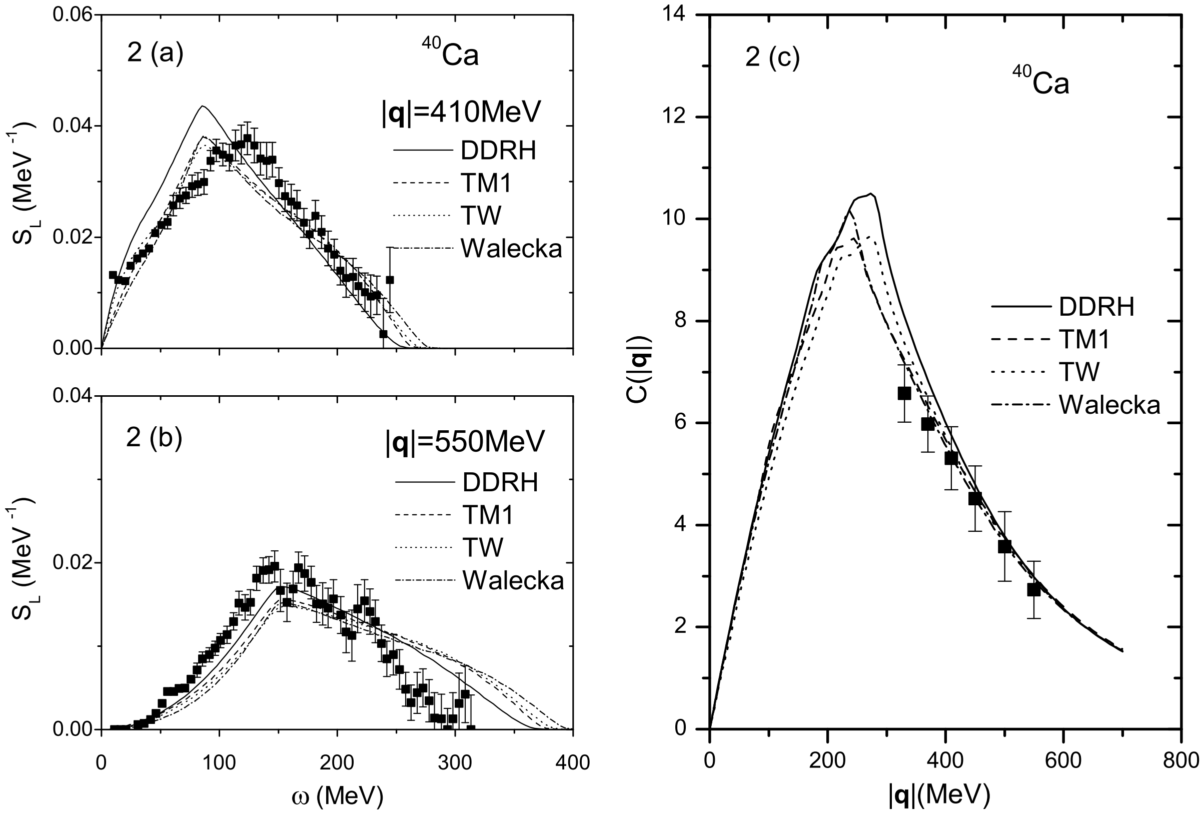

Fig. 2. Longitudinal response function and Coulomb sum rule for ^{40}Ca. Experimental data are taken from ref. [14].

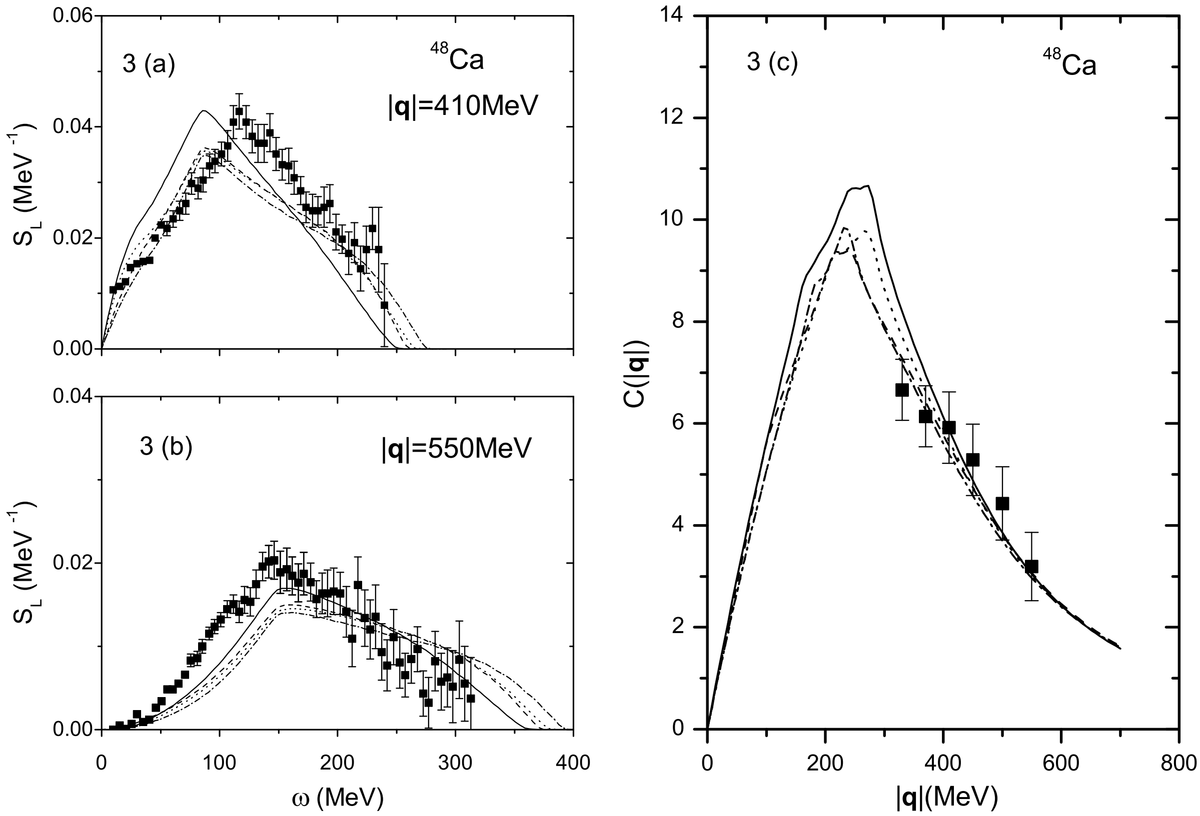

Fig. 3. Same as fig. 2, but for ^{48}Ca.

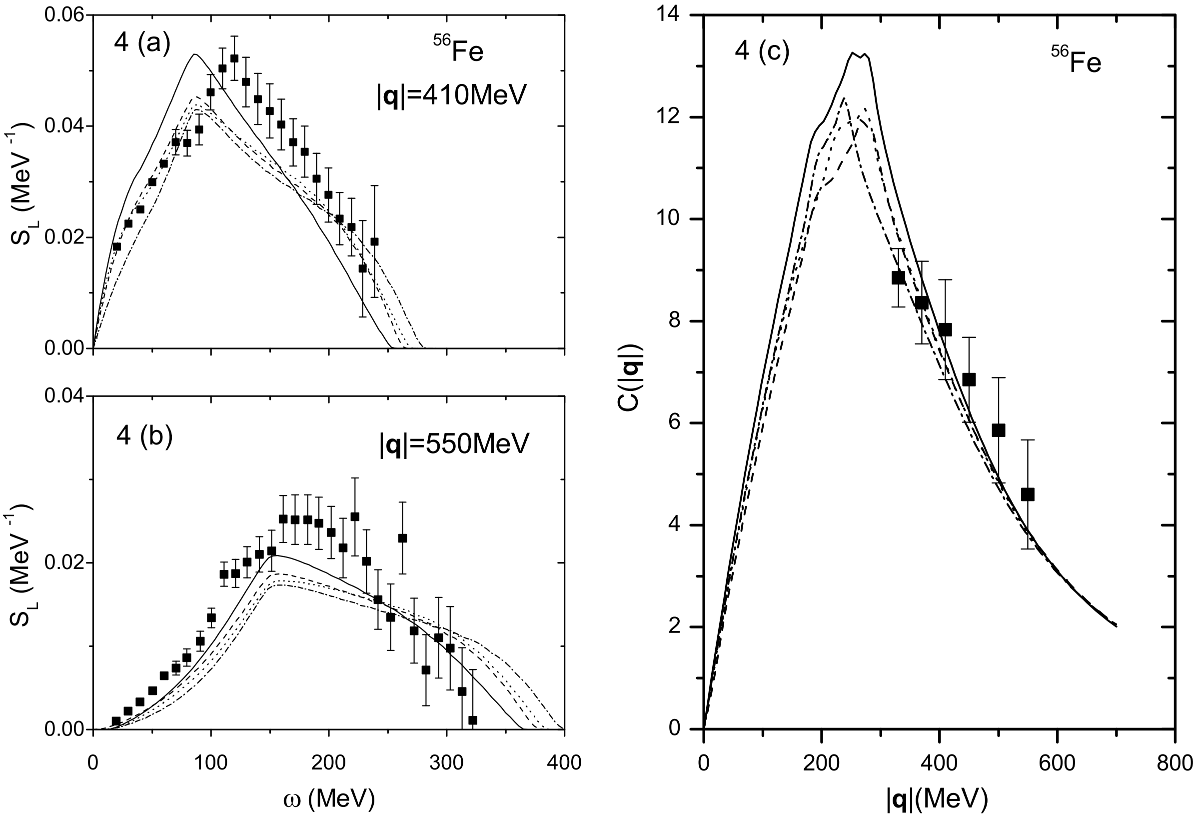

Fig. 4. Same as fig. 2, but for ^{56}Fe.

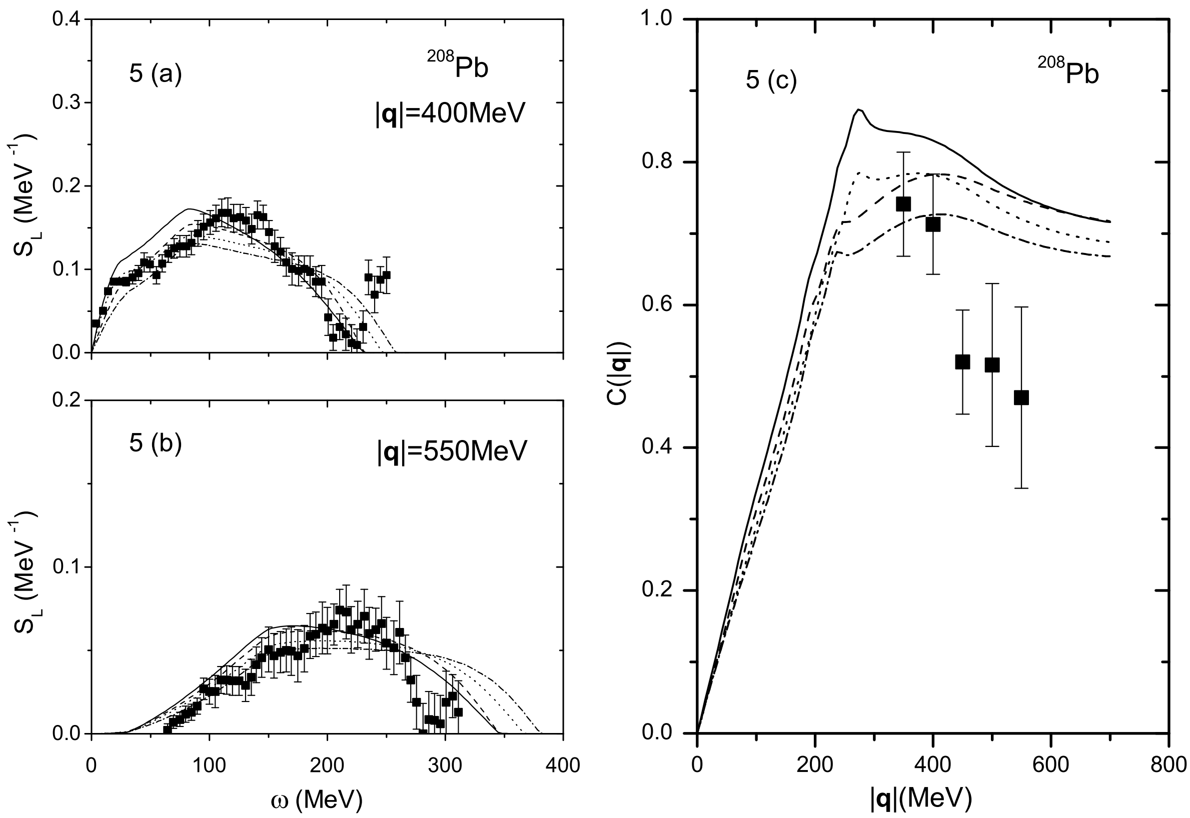

Fig. 5. Longitudinal response function and Coulomb sum rule for ^{208}Pb. Experimental data are taken from ref. [17].

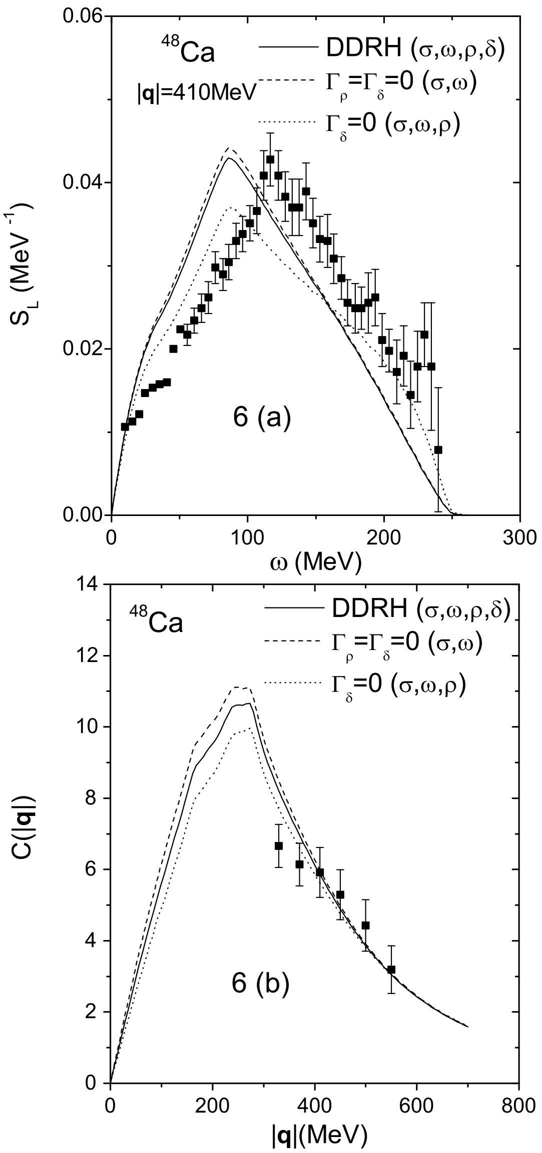

Fig. 6. Longitudinal response function (panel 6(a)) and Coulomb sum rule (panel 6(b)) for ^{48}Ca. Experimental data are taken from ref. [14].

the maximum value of the curve with the σ, ω, ρ, and δ mesons (solid line) lies between those with σ and ω mesons (broken line) and with the σ, ω, and ρ mesons (dotted line). This result is caused by the competition of the isospin-dependent attractive (the δ-meson) and repulsive (the ρ-meson) contributions. The net contribution including both ρ and δ mesons cancels out. The roles of both ρ and δ mesons are also shown in fig. 6(b). The curve with the ρ-meson (*i.e.*, σ, ω, ρ labelled by dotted line) becomes lower due to the isospin-dependent repulsive contribution of the ρ-meson, while the curve without the ρ-meson (*i.e.*, σ, ω labelled by broken line) is higher. When the δ-meson is added, it is shown that the curve in the DDRH model (*i.e.*, σ, ω, ρ, δ labelled by solid line) lies between those of curves with (dotted line) and without (broken line) the ρ-meson.

4 Summary

The DDRH model is extended to study quasielastic electron scattering, and the obtained results in the DDRH model are compared with those in other ones. It has been shown that the meson effective masses due to the nonlinear terms in the nonlinear Walecka model should be used to obtain the meson Green's functions when the longitudinal response function and the Coulomb sum rule are calculated. The process of quasielastic electron scattering can be described well in the various relativistic mean-field models except for the Coulomb sum rule in a heavy nucleus like ^{208}Pb. Finally, the roles of both δ and ρ mesons that play in quasielastic electron scattering are tested in the DDRH model. The isospin-dependent attractive (the δ-meson) and repulsive (the ρ-meson) properties are clearly shown in quasielastic electron scattering.

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